In a nutshell: Newton's method in *n* dimensions

Given a continuous and differentiable vector-valued function **f** of a vector variable with one initial approximation of a root $\mathbf{x}_0$ where the Jacobian at that point $\mathbf{J}(\mathbf{f})(\mathbf{x}_0)$ is invertible. If the value is already zero, we have already found a root. This algorithm uses iteration, Taylor series and solving systems of linear equations to approximate a root.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the root cannot exceed this value.
ε_{abs}	The value of the function at the approximation of the root cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. Solve $\mathbf{J}(\mathbf{f})(\mathbf{x}_k)\Delta\mathbf{x}_k = -\mathbf{f}(\mathbf{x}_k)$ for $\Delta\mathbf{x}_k$ where $\mathbf{J}(\mathbf{f})(\mathbf{x})$ is the Jacobian of **f** evaluated at the point **x**.
   Let $\mathbf{x}_{k+1} \leftarrow \mathbf{x}_k + \Delta\mathbf{x}_k$.
   a. If $\mathbf{x}_{k+1}$ has any entries that are not finite floating-point numbers, return signalling a failure to converge.
   b. If $\|\mathbf{x}_{k+1} - \mathbf{x}_k\|_2 < \varepsilon_{\text{step}}$ and $\|\mathbf{f}(\mathbf{x}_{k+1})\|_2 < \varepsilon_{\text{abs}}$, return $\mathbf{x}_{k+1}$.
4. Increment k and return to Step 2.

Convergence

If h is the error, it can be show that the error decreases according to $O(h^2)$. This technique is not guaranteed to converge if there is a root, for the Jacobian could be close to singular, causing the next approximation to be arbitrarily far from the previous approximation.